

MSE200-002, Chapter 8

Homework solutions

8.2. a) ice and water; b) water and vapor; 3) water, ice and vapor.

8.3. There are 3 triple point lines.

The lower one: vapor + α Fe + γ Fe

The middle one: vapor + γ Fe + δ Fe

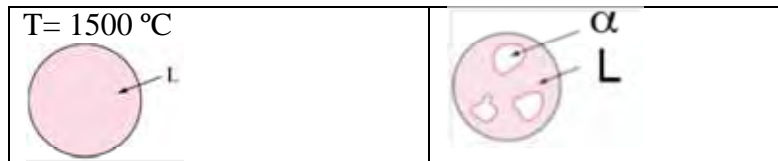
The upper one: vapor + δ Fe + $\lambda\theta\upsilon\iota\delta$

8.4. Gibbs phase rule: $P + F = C+2$, with varying pressure; $P+F = C+1$, with constant pressure

8.5. a) $F=0$, b) $F=2$

8.6. A system with two components (elements) with unlimited solubility in each other.

8.10. a) at 1350 °C. i) $L + \alpha$; ii) C_L : 59% Ni, $C_\alpha = 72\%$ Ni;
 iii) $C_0 = 70\%$ Ni. Fraction of L phase, $X_L = (C_\alpha - C_0)/(C_\alpha - C_L) = 15\%$
 Fraction of a phase, $X_\alpha = 1 - X_L = 85\%$
 b) at 1500 °C, Only liquid phase exist, with a composition of 70%.
 c)



8.15. $C_0 = 88\%$

(a) $T = 1000\text{ °C}$, (i) L, (ii) $C_L = 88\%$ Ag, (iii) 100% L,

(b) $T = 800\text{ °C}$, (i) $L + \beta$, (ii) $C_L = 78\%$ Ag, $C_\beta = 93\%$ Ag,

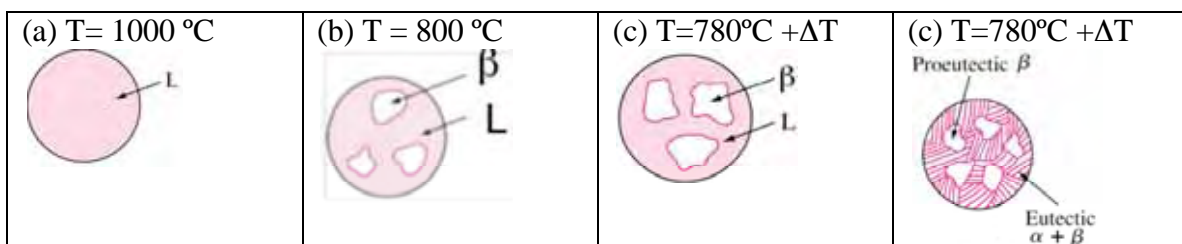
(iii) $X_L = (93\% - 88\%)/(93\% - 78\%) = 33.3\%$, $X_\beta = 1 - X_L = 66.7\%$

(c) $T = 780\text{ °C} + \Delta T$, (i) $L + \beta$, (ii) $C_L = 71.9\%$ Ag, $C_\beta = 91.2\%$ Ag,

(iii) $X_L = (91.2\% - 88\%)/(91.2\% - 71.9\%) = 16.2\%$, $X_\beta = 1 - X_L = 83.4\%$

(d) $T = 780\text{ °C} - \Delta T$, (i) $\alpha + \beta$, (ii) $C_\alpha = 7.9\%$ Ag, $C_\beta = 91.2\%$ Ag,

(iii) $X_\alpha = (91.2\% - 88\%)/(91.2\% - 7.9\%) = 3.8\%$, $X_\beta = 1 - X_\alpha = 96.2\%$



8.16. $C_0 = 40\%$, $W_0 = 500\text{g}$ (a) $T = 850^\circ\text{C}$, $L + \alpha$, $C_L = 55\%$, $C_\alpha = 7.8\%$,

$$X_L = (40\% - 7.8\%) / (55\% - 7.8\%) = 68.2\%, \quad X_\beta = 1 - X_L = 31.8\%$$

$$W_L = W_0 \times X_L = 500\text{g} \times 68.2\% = 341\text{g}, \quad W_\alpha = W_0 \times X_\alpha = 500\text{g} \times 68.2\% = 159\text{g}$$

(b) $T = 780^\circ\text{C} + \Delta T$, $L + \alpha$, $C_L = 71.9\%$, $C_\alpha = 7.9\%$,

$$X_L = (40\% - 7.9\%) / (71.9\% - 7.9\%) = 50.2\%, \quad X_\beta = 1 - X_L = 49.8\%$$

$$W_L = W_0 \times X_L = 500\text{g} \times 50.2\% = 251\text{g}, \quad W_\alpha = W_0 \times X_\alpha = 500\text{g} \times 49.8\% = 249\text{g}$$

(c) $T = 780^\circ\text{C} - \Delta T$, $\alpha + \beta$, $C_\alpha = 7.9\%$, $C_\beta = 91.2\%$ Ag

$$X_\alpha = (91.2\% - 40\%) / (91.2\% - 7.9\%) = 61.5\%,$$

$$W_\alpha = W_0 \times X_\alpha = 500\text{g} \times 61.5\% = 307\text{g}$$

(d) $X_\beta = 1 - X_\alpha = 38.5\%$

$$W_\beta = W_0 \times X_\beta = 500\text{g} \times 38.5\% = 193\text{g}$$

8.17. $T = 183^\circ\text{C} - \Delta T$, $X_{\text{proeutectic } \beta} = 60\%$, $X_{\text{eutectic}} = 40\%$ (Corrected), $C_{\text{eutectic}} = 61.9\%$,

$C_\beta = 97.5\%$, find C_0

$$X_{\text{proeutectic } \beta} = 60\% = (C_0 - C_{\text{eutectic}}) / (C_\beta - C_{\text{eutectic}}) = (C_0 - 61.9\%) / (97.5\% - 61.9\%)$$

$$C_0 = 83.26\%$$

8.18. $T = 50^\circ\text{C}$, $X_\beta = 40\%$, $X_\alpha = 60\%$, $C_\alpha = 3\%$, $C_\beta = 100\%$, find C_0

$$X_\beta = 40\% = (C_0 - C_\alpha) / (C_\beta - C_\alpha) = (C_0 - 3\%) / (100\% - 3\%)$$

$$C_0 = 41.8\%$$

8.19. $C_0 = 70\%$ Sn,

(a) hyper eutectic, (b) 98% Sn,

(c) at $T = 183^\circ\text{C} + \Delta T$, $C_L = 61.9\%$, $C_\beta = 97.5\%$,

$$X_L = (97.5\% - 70\%) / (97.5\% - 61.9\%) = 77.2\%, \quad X_\beta = 1 - X_L = 2.8\%$$

(d) at $T = 183^\circ\text{C} - \Delta T$, $C_\alpha = 19.2\%$, $C_\beta = 97.5\%$,

$$X_\alpha = (97.5\% - 70\%) / (97.5\% - 19.2\%) = 35.1\%, \quad X_\beta = 1 - X_L = 64.9\%$$

(e) $C_\alpha = 1\%$, $C_\beta = 100\%$,

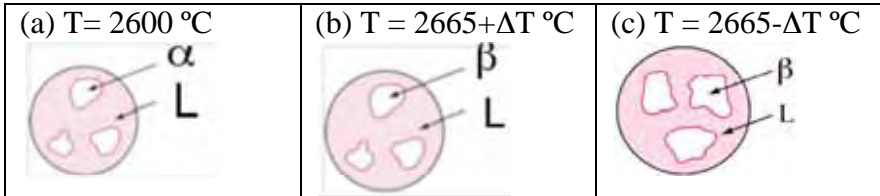
$$X_\alpha = (100\% - 70\%) / (100\% - 1\%) = 30.3\%, \quad X_\beta = 1 - X_L = 69.7\%$$

8.20. $C_0 = 30\text{ wt}\%$ Os(a) at $T = 2600^\circ\text{C}$, (i) $L + \alpha$, (ii) $C_L = 17\%$, $C_\alpha = 39\%$,

$$\text{(iii) } X_L = (39\% - 30\%) / (39\% - 17\%) = 41.0\%, \quad X_\alpha = 1 - X_L = 59.0\%$$

(b) at $T = 2665^\circ\text{C} + \Delta T$, (i) $L + \beta$, (ii) $C_L = 23.0\%$, $C_\beta = 61.5\%$,
 (iii) $X_L = (61.5\% - 30\%) / (61.5\% - 23\%) = 81.8\%$, $X_\beta = 1 - X_L = 18.2\%$

(c) at $T = 2665^\circ\text{C} - \Delta T$, (i) $L + \alpha$, (ii) $C_L = 23.0\%$, $C_\alpha = 43.0\%$,
 (iii) $X_L = (43\% - 30\%) / (43\% - 23\%) = 65\%$, $X_\beta = 1 - X_L = 35\%$



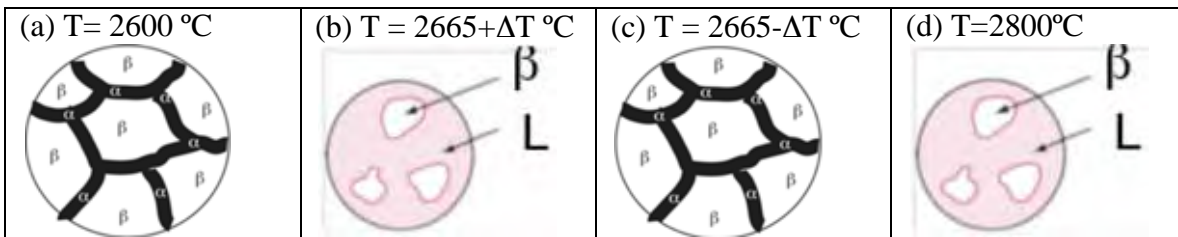
8.21. $C_0 = 60 \text{ wt}\% \text{ Os}$

(a) at $T = 2600^\circ\text{C}$, (i) $\alpha + \beta$, (ii) $C_\alpha = 42.5\%$, $C_\beta = 61.4\%$,
 (iii) $X_\alpha = (61.4\% - 60\%) / (61.4\% - 42.5\%) = 7.4\%$, $X_\beta = 1 - X_\alpha = 92.6\%$

(b) at $T = 2665^\circ\text{C} + \Delta T$, (i) $L + \beta$, (ii) $C_L = 23.0\%$, $C_\beta = 61.5\%$,
 (iii) $X_L = (61.5\% - 60\%) / (61.5\% - 23\%) = 3.9\%$, $X_\beta = 1 - X_L = 96.1\%$

(c) at $T = 2665^\circ\text{C} - \Delta T$, (i) $\alpha + \beta$, (ii) $C_\alpha = 43.0\%$, $C_\beta = 61.5\%$,
 (iii) $X_\alpha = (61.5\% - 60\%) / (61.5\% - 43.0\%) = 8.1\%$, $X_\beta = 1 - X_\alpha = 91.9\%$

(d) $T = 2800^\circ\text{C} + \Delta T$, (i) $L + \beta$, (ii) $C_L = 45\%$, $C_\beta = 87\%$,
 (iii) $X_L = (87\% - 60\%) / (87\% - 45\%) = 40.5\%$, $X_\beta = 1 - X_L = 59.5\%$



8.24. $C_0 = 4.2\%$. $C_\alpha = 4.0\%$, $C_L = 5.4\%$. At the peritectic temperature (1517°C), the weight% of alloy solidifies equals the L% left just before the peritectic reaction.
 $X_L = (4.2\% - 4.0\%) / (5.4\% - 4.0\%) = 14\%$

8.26. $C_0 = 4.2\%$. $C_\alpha = 4.0\%$, $C_L = 5.4\%$. At the $1517^\circ\text{C} + \Delta T$, there are two phases: $\alpha + L$.
 From 8.24: $X_L = 14\%$, $X_\alpha = 1 - X_L = 86\%$

8.29. $C_0 = 10\% \text{ Pb}$,

(a) at $T=100^\circ\text{C}$, two phases: pure Cu, $C_{\text{Cu}} = 0\% \text{ Pb}$, and Pb: $C_{\text{Pb}}=100\% \text{ Pb}$,

$$X_{\text{Pb}} = (10\%-0)/(100\%-0) = 10\%, X_{\text{Cu}} = 1-X_{\text{Pb}} = 90\%$$

(b) (a) at $T=955 + \Delta T^\circ\text{C}$, two phases: $\alpha + L_1$, $C_\alpha = 0\% \text{ Pb}$, and Pb: $C_{L_1}=36\% \text{ Pb}$,

$$X_{L_1} = (10\%-0)/(36\%-0) = 27.8\%, X_\alpha = 1-X_{L_1} = 72.2\%$$

(c) (a) at $T=955 - \Delta T^\circ\text{C}$, two phases: $\alpha + L_2$, $C_\alpha = 0\% \text{ Pb}$, and Pb: $C_{L_2}=87\% \text{ Pb}$,

$$X_{L_2} = (10\%-0)/(87\%-0) = 11.5\%, X_\alpha = 1-X_{L_2} = 88.5\%$$

(d) at $T=200^\circ\text{C}$. The same as at $T=100^\circ\text{C}$.

8.31. at $T=955 + \Delta T^\circ\text{C}$, two phases: $\alpha + L_1$, $C_\alpha = 0\% \text{ Pb}$, and Pb: $C_{L_1}=36\% \text{ Pb}$,

$X_{L_1} = 30\%$, find C_0

$$X_{L_1} = (C_0-0)/(36\%-0) = 30\%, C_0 = 10.8\%$$